



Analytical model of the pulse tube engine



Stefan Moldenhauer

Institute of Thermodynamics and Fluid Mechanics, Ilmenau University of Technology, P.O. Box 100565, 98684 Ilmenau, Germany

ARTICLE INFO

Article history:

Received 12 March 2013

Received in revised form

19 August 2013

Accepted 21 August 2013

Available online 11 October 2013

Keywords:

Pulse tube engine

Heat engine

Energy conversion

Waste heat

Thermoacoustics

Analytical analysis

ABSTRACT

The pulse tube engine represents the thermodynamic inversion of the pulse tube refrigerator used in cryogenic cooling applications. It has a high potential to be used as a prime mover for the conversion of low grade waste heat into mechanical or electrical energy. This paper describes an analytical analysis of the pulse tube engine based on a zero-dimensional model. During compression and expansion, the engine components are considered as isothermal with characteristic temperatures. At the piston's dead centers, a thermal relaxation model is used to switch between these temperatures. Analytical relations for the pV -work developed by the pulse tube engine and its efficiency are derived. The irreversible nature of the pulse tube engine is studied by calculating the entropy production in the components. Furthermore, the thermodynamic cycle is investigated analytically under variation of design features and operating conditions. The results are compared to prior numerical studies. The minimal temperature ratio above which the engine provides a work output is derived analytically and compared to experimental observations. Fundamental characteristics and application limitations of the pulse tube engine are disclosed. An upper limit for the efficiency of the pulse tube engine is derived theoretically and confirmed experimentally as well as through numerical calculations.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

In the 1960's Gifford and Longworth invented the basic pulse tube refrigerator [1]. These coolers have been research and development objects for the last 40 years, e.g. Refs. [2,3], and are employed in cryogenic technologies to provide as one-stage refrigerators temperatures around the condensation temperature of nitrogen (77.2 K at atmospheric pressure) or as two-stage refrigerators temperatures down to the condensation temperature of helium (4.2 K at atmospheric pressure).

The reversing of the pulse tube process enables the design of a simple prime mover – the pulse tube engine – for the conversion of externally supplied thermal energy into mechanical energy or, using an electrical generator, into electrical energy.

The basic engine design is shown in Fig. 1. A compression/expansion cylinder supplies a pressure swing and gas shifting to the connected system consisting of an almost adiabatic buffer space (pulse tube) and an isothermalizer (regenerator). Both components are fluidically linked via a gas heater (h_{hx}) at the temperature T_h , providing the heat input. A gas cooler (ch_{x1}) at the temperature T_c is located between cylinder and pulse tube. A heat sink (ch_{x2}) at the temperature T_c – to set the depicted material temperature distribution – is located at the end of the regenerator. For the sake of

simplification, it is assumed that the material temperature is spatially and temporally constant in the heat exchangers and has the shown temporal constant, linear distributions in the pulse tube and regenerator. According to the numerical and experimental results of [4], the linear material temperature distribution in the regenerator is a good assumption, whereas the material temperature distribution of the pulse tube is slightly non-linear. This non-linearity has only a second order influence on the heat transfer conditions in the pulse tube, and is thus neglected.

The conversion of compression/expansion work (pV -work) into mechanical work and vice versa is carried out by the resulting pressure force on the piston within the cylinder, which is located at cold temperature level. For the conversion of mechanical energy into electrical energy, an electrical generator can be joined to the cylinder/piston configuration via a crank gear mechanism.

The mode of operation of the pulse tube engine is based on a simultaneous compression and moving of the working gas into an area of high material temperature followed by a simultaneous expansion and moving of the working gas into an area of low material temperature. Therefore, the piston moving from its bottom dead center (BDC) to its top dead center (TDC) compresses the working gas and pushes gas from the cylinder via ch_{x1} into the cold side of the pulse tube. Passing ch_{x1}, the gas is cooled down to T_c . The situation is illustrated in Fig. 1 by the dotted gas temperature distribution in the pulse tube during the compression stroke. Meanwhile, in the pulse tube, gas is compressed and shifted from the

E-mail address: stefan.moldenhauer@tu-ilmenau.de.

List of symbols

$\dot{H}$	enthalpy flux	$S_{\text{irr,reg}}$	entropy production regenerator
$\dot{m}$	mass flux	T	temperature
$\dot{m}_{\text{chx}}$	mass flux via chx	T^0	temperature of reference state
$\dot{m}_{\text{hhx}}$	mass flux via hhx	T_c	heat sink temperature
$\dot{Q}_{\text{hx}}$	gas-material heat flux	T_h	heat source temperature
$\dot{S}$	entropy flux	T_{chx}	gas temperature chx
$\dot{S}_{\text{hx}}$	gas-material entropy flux	T_{cyl}	gas temperature cylinder
W	compression/expansion (pV –) power	T_{hhx}	gas temperature hhx
c_1	fitting parameter	$T_{\text{m,chx}_1}$	material temperature chx ₁
C_c	compression constant	$T_{\text{m,chx}_2}$	material temperature chx ₂
C_e	expansion constant	$T_{\text{m,cyl}}$	material temperature cylinder
C_{hp}	high pressure constant	$T_{\text{m,hhx}}$	material temperature hhx
C_{lp}	low pressure constant	$T_{\text{m,pt}}$	material temperature pulse tube
f	operating frequency	$T_{\text{m,reg}}$	material temperature regenerator
L_{pt}	pulse tube length	$T_{\text{pt,c}}$	compression gas temperature pulse tube
L_{reg}	regenerator length	$T_{\text{pt,e}}$	expansion gas temperature pulse tube
m	gas mass	T_{pt}	gas temperature pulse tube
p^0	pressure of reference state	T_{reg}	gas temperature regenerator
p_c	compression pressure	U	internal energy gas
p_e	expansion pressure	V^0	volume of reference state
p_f	filling pressure	V_d	displacement volume
p_g	gross power	V_{hx}	total heat exchanger gas volume
P_n	net power	V_{max}	maximum engine gas volume
$P_{\text{l,int}}$	internal power loss	V_{min}	minimum engine gas volume
p_{max}	maximum pressure	V_{pt}	pulse tube gas volume
p_{min}	minimum pressure	V_{reg}	regenerator gas volume
Q_{chx}	cycle heat chx	W_c	compression work
Q_{cyl}	cycle heat cylinder	W_e	expansion work
Q_{hhx}	cycle heat hhx	W_{pV}	pV –work
Q_{in}	supplied heat	x_{pt}	pulse tube position
Q_{pt}	cycle heat pulse tube	x_{reg}	regenerator position
Q_{reg}	cycle heat regenerator	F	fitting parameter
r_c	compression ratio	ε	compression/expansion factor
r_d	distribution ratio	ε_c	compression factor
S	gas entropy	ε_e	expansion factor
$S_{\text{irr,chx}}$	entropy production chx	η_{th}	thermal efficiency
$S_{\text{irr,cyl}}$	entropy production cylinder	λ	boundary condition factor
$S_{\text{irr,hhx}}$	entropy production hhx	Π	pressure ratio
$S_{\text{irr,pt}}$	entropy production pulse tube	σ	sign function
		τ	temperature ratio
		τ_{opt}	optimal temperature ratio

cold to the hot side of the tube. Gas leaving the pulse tube at its hot side via hhx takes on heat and enters the hot side of the regenerator with the temperature T_h . The regenerator material, having the depicted material temperature distribution of Fig. 1, absorbs heat from the working gas. Thus, the heat taken from hhx is entirely given off to the regenerator material. The thermal capacity of the regenerator material is high enough to experience no significant temperature swing. Since cold gas was shifted during compression from the cold to the hot side of the pulse tube, it is able to take on

heat from the tube's wall material while the piston is moving around TDC (high pressure thermal relaxation). The pressure increases slightly. Now the piston moves from TDC back to BDC which results in an expansion of the working gas flowing from the regenerator via hhx into the hot side of the pulse tube. The situation is illustrated in Fig. 1 by the dotted gas temperature distribution in the pulse tube during the expansion stroke. The regenerator material gives back the heat stored during the compression phase. Since the gas entering hhx is already at T_h , it receives only an

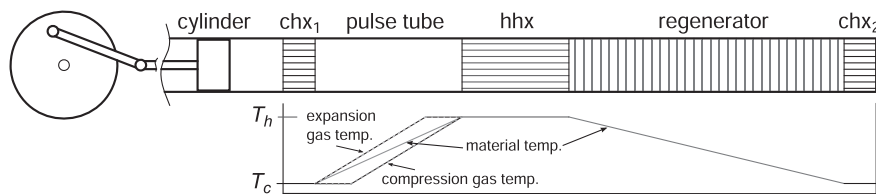


Fig. 1. Basic design of the pulse tube engine. chx₁ – cold heat exchanger 1, chx₂ – cold heat exchanger 2, hhx – hot heat exchanger. For the sake of simplification, the material temperature distribution is assumed to be linear in the pulse tube and the regenerator as well as spatial constant in the other components. The ideally assumed gas temperature distribution in the pulse tube during compression and expansion is illustrated using dotted lines.

insignificant amount of heat. Meanwhile, in the pulse tube, gas is expanded and shifted from the hot to the cold side of the tube. Gas leaving the pulse tube at its cold side via chx_1 is cooled and enters the cylinder with the temperature T_c . In the cylinder the gas is further expanded and its temperature goes below T_c . Since hot gas was shifted during expansion from the hot to the cold side of the pulse tube, it is able to give off heat to the tube's wall material while the piston is moving around BDC (low pressure thermal relaxation). The pressure decreases slightly till the initial state is reached again. Absorbing heat at high pressure and rejecting heat at low pressure results in a higher mean pressure during expansion than during compression and in a release of pV -work.

First concepts of prime movers similar to the pulse tube engine were presented in 1987 by Chen and West [5]. In 1995, Tailer was granted a US patent [6] for a heat engine based on a phase delay between pressure fluctuations and heat transfer (thermal lag [7,8]). A patent describing a device resembling the pulse tube engine was published in 2000 by Hofmann [9]. The concept of the pulse tube engine was first presented in its entirety by Hamaguchi et al. [10] in 2005, and later by Organ [11] in 2007. A number of experimental investigations to characterize this type of prime mover were also published by Hamaguchi et al. [10,12].

According to work flux measurements of Yoshida et al. [13] in 2009, the pulse tube engine can be classified into the thermoacoustic [14] standing wave engine group [15–18] with an underlying intrinsically irreversible thermodynamic cycle [19], although the pulse tube engine is – by comparing its working principle with the thermoacoustic engines investigated in Refs. [20–22] – analogous to the Stirling engine, not a thermoacoustic device.

In contrast to the Stirling engine, the pulse tube engine has no displacer for the provision of the phase delay between pressure fluctuation and volume flux, which is operationally necessary. In similarity with the pulse tube refrigerator, the gas dynamics and heat transfer conditions in the pulse tube and the regenerator enable the generation of this phase delay. Thus, the pulse tube engine operates with only one piston. Due to the difficulty in adjusting the mentioned phase delay by the correct assembly of pulse tube and regenerator, the design of the pulse tube engine is bound to a deep understanding of the physical phenomena related to its operation. In order to meet this challenge, a comprehensive thermodynamic analysis using a numerical model of the pulse tube engine was performed by Moldenhauer et al. in 2011 and published in 2013 [23]. There, it was shown that a broken thermodynamic symmetry between the pulse tube and the regenerator is the underlying working principle of the pulse tube engine. Thermodynamic asymmetry results from the fluidic linkage of the almost adiabatic pulse tube and the regenerator. Thus – in contrast to the Stirling engine – the regenerator is essential for the operation of the pulse tube engine. The regenerator of the pulse tube engine can be regarded as an isothermalizer, which together with the almost adiabatic pulse tube creates a thermodynamic asymmetry.

An extended version of the numerical model of [23] was presented by Moldenhauer et al. in 2013 [24]. The numerical results are compared to experimental performance data of a pulse tube engine, built using pressurized Helium as working fluid. The numerical model presented in Ref. [24] can be used to design a working pulse tube engine and predict its performance.

Although the pulse tube engine is thermodynamically similar to the pulse tube refrigerator, its theory is still much less understood than for its cryogenic counterpart. To the best of the author's knowledge, no closed solution of the pulse tube engine's governing equations has been obtained yet. In order to bridge this gap, the present work represents a contribution to the theory of the pulse tube engine by the development of an analytical model for a closed solution of the underlying differential

equations. The aim of this research is to determine the basic physical characteristics of the pulse tube engine, to find analytical relations for the developed pV -work and its efficiency as well as performance limitations. The irreversible nature of the pulse tube engine is studied by calculating the entropy production in the components. Furthermore, the thermodynamic cycle is investigated under variation of design features and operating conditions. Finally, the analytical results are compared to numerical and experimental results obtained from the numerical models given in Refs. [23,24] and the working pulse tube engine presented in Refs. [4,24].

2. Governing equations

The system of differential equations underlying the pulse tube engine is based on a control volume analysis [25,26]. Each component of the engine is regarded as one control volume with one representative value of each physical property depending only on time. The balances for mass, energy, and entropy of the working gas lead to the governing equations for the time-dependent physical properties of the working gas. The change in the internal energy of the material is disregarded. Since pulse tube engines are typically operating at low frequencies, a momentum balance is not performed.

Fig. 2 shows the schematic representation of the control volume V . The internal energy of the gas is represented by U . The gas mass content of the control volume is m and its entropy S . The material has the constant temperature T_m . Heat can be exchanged between the engine material and the working gas via the heat flux $\dot{Q}_{hx}$. The entropy flux due to heat transfer between gas and material is represented by $\dot{S}_{hx}$. The working gas in the cylinder is charged with the compression/expansion power (pV -power) $\dot{W}$.

Between the control volumes, mass fluxes $\dot{m}$, enthalpy fluxes $\dot{H}$, and entropy fluxes $\dot{S}$ are exchanged. Fluxes at the control volume's left side are marked with ' and fluxes at the control volume's right side with '. Heat and entropy fluxes due to heat conduction are not considered. All entering fluxes are given a positive sign.

The resulting system of differential equations for the pressure p , the gas temperature T , and the entropy production rate dS_{irr}/dt in each engine component is given by

$$\frac{dp}{dt} = \frac{R_s T}{V} (\dot{m}' + \dot{m}'') - \frac{p}{V} \frac{dV}{dt} + \frac{p}{T} \frac{dT}{dt}, \quad (1)$$

$$\frac{c_v p V}{R_s T} \frac{dT}{dt} = -c_v T (\dot{m}' + \dot{m}'') + \dot{Q}_{hx} - p \frac{dV}{dt} + \dot{m}' c_p T' + \dot{m}'' c_p T'', \quad (2)$$

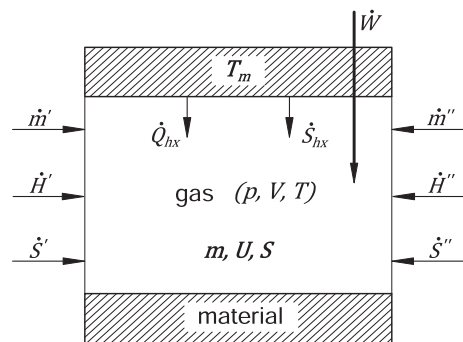


Fig. 2. Schematic representation of the control volume with the internal energy of the gas U , the gas mass m , and the gas entropy S . Heat and entropy exchanged between the gas and the material are represented by $\dot{Q}_{hx}$ and $\dot{S}_{hx}$. Mass, enthalpy, and entropy fluxes between the control volumes are denoted by $\dot{m}$, $\dot{H}$, and $\dot{S}$. Left and right side fluxes are marked with ' and '. The compression/expansion power is $\dot{W}$ and the material temperature is T_m .

and

$$\begin{aligned} \frac{dS_{\text{irr}}}{dt} = & (\dot{m}' + \dot{m}'') \left(c_v \left(\ln \frac{T}{T^0} - 1 \right) + R_s \ln \frac{V}{V^0} \right) \\ & + \dot{m}' \left(c_p \left(\frac{T'}{T} - \ln \frac{T'}{T^0} \right) + R_s \ln \frac{p'}{p^0} \right) \\ & + \dot{m}'' \left(c_p \left(\frac{T''}{T} - \ln \frac{T''}{T^0} \right) + R_s \ln \frac{p''}{p^0} \right), \end{aligned} \quad (3)$$

where T^0 , p^0 , and V^0 are the physical properties of the gas at the reference state.

3. Parameter definitions

A generalized characterization of the pulse tube engine process requires the definition of dimensionless operating and design parameters.

The operating parameters are the **temperature ratio** τ , defined as the ratio between the hot temperature T_h and the cold temperature T_c

$$\tau := \frac{T_h}{T_c} \quad (4)$$

as well as the **pressure ratio** Π , representing the ratio between maximum pressure p_{max} and minimum pressure p_{min}

$$\Pi := \frac{p_{\text{max}}}{p_{\text{min}}} \quad (5)$$

Design parameters are the **distribution ratio** r_d , calculated as the ratio between the regenerator gas volume V_{reg} and the pulse tube gas volume V_{pt}

$$r_d := \frac{V_{\text{reg}}}{V_{\text{pt}}} \quad (6)$$

as well as the **compression ratio** r_c , representing the ratio between the maximum engine gas volume V_{max} and the minimum engine gas volume V_{min} minus one

$$\begin{aligned} r_c &:= \frac{V_{\text{max}}}{V_{\text{min}}} - 1 = \frac{V_{\text{pt}} + V_{\text{reg}} + V_{\text{hx}} + V_d}{V_{\text{pt}} + V_{\text{reg}} + V_{\text{hx}}} - 1 \\ &= \frac{V_d}{V_{\text{pt}} + V_{\text{reg}} + V_{\text{hx}}} \end{aligned} \quad (7)$$

The displacement volume of the cylinder and the sum of the heat exchanger gas volumes are denoted with V_d and V_{hx} , respectively.

4. Analytical model of the pulse tube engine

Pulse tube engines typically operate at low frequencies of only a few Hz. Such slow thermodynamic processes can be approached using an isothermal model [27]. An isothermal analysis ($dT/dt = 0$) of the thermodynamic cycle decouples the system of differential equations (1), (2), and (3) and enables an analytical integration of the decoupled differential equations.

4.1. Isothermal approach

As already shown in Fig. 1, the cylinder has the uniform material temperature $T_{\text{m,cyl}} = T_c$, the two cold heat exchangers have the uniform material temperatures $T_{\text{m,chs}_1} = T_{\text{m,chs}_2} = T_c$, and the hot

heat exchanger has the uniform material temperature $T_{\text{m,hhx}} = T_h$. Pulse tube and regenerator have the space-dependent fixed linear material temperature distributions

$$T_{\text{m,pt}}(x_{\text{pt}}) = T_c + \frac{x_{\text{pt}}}{L_{\text{pt}}}(T_h - T_c)$$

and

$$T_{\text{m,reg}}(x_{\text{reg}}) = T_h - \frac{x_{\text{reg}}}{L_{\text{reg}}}(T_h - T_c),$$

with x_{pt} denoting the pulse tube position starting at its cold side with $x_{\text{pt}} = 0$ and x_{reg} denoting the regenerator position starting at its hot side with $x_{\text{reg}} = 0$. The length of the pulse tube is L_{pt} and of the regenerator L_{reg} .

The gas volumes of the heat exchangers are neglected. Thus, the cold heat exchanger chs_2 has not been considered in this analysis. For pulse tube and regenerator, the (spatial) mean material temperatures are given by

$$\langle T_{\text{m,pt}} \rangle = \langle T_{\text{m,reg}} \rangle = \frac{1}{2}(T_h + T_c) = \frac{T_c}{2}(\tau + 1).$$

In the isothermal analysis of the pulse tube engine the gas in the cylinder, the heat exchangers, and the regenerator is always in thermal equilibrium with the material [28]. The constant gas temperatures in these components are $T_{\text{cyl}} = T_{\text{g,cyl}} = T_c$, $T_{\text{chs}} = T_{\text{g,chs}_1} = T_c$, $T_{\text{hhx}} = T_{\text{g,hhx}} = T_h$, and $T_{\text{reg}} = T_{\text{g,reg}} = \langle T_{\text{m,reg}} \rangle = T_c(\tau + 1)/2$.

Due to the cold gas being pushed into the cold side of the pulse tube during the compression stroke, the pulse tube's mean gas temperature is below its wall material temperature while the compression takes place. During the expansion stroke, hot gas is pushed into the hot side of the pulse tube. Thus its mean gas temperature is above the wall material temperature while the expansion takes place. The situation is illustrated in Fig. 3. For compression and expansion, it is assumed that the gas in the pulse tube has a constant temperature slightly below the mean wall material temperature (compression stroke) respectively slightly above the mean wall material temperature (expansion stroke).

Using this assumption, the pulse tube's gas temperature is set to

$$T_{\text{pt,c}} = (1 - \varepsilon_c) \langle T_{\text{m,pt}} \rangle = (1 - \varepsilon_c) \frac{T_c}{2}(\tau + 1) \quad (8)$$

during the compression stroke and

$$T_{\text{pt,e}} = (1 + \varepsilon_e) \langle T_{\text{m,pt}} \rangle = (1 + \varepsilon_e) \frac{T_c}{2}(\tau + 1) \quad (9)$$

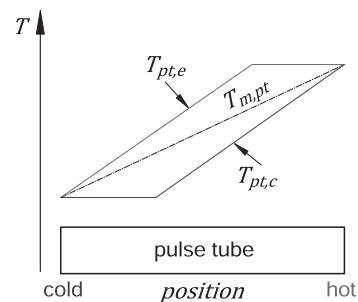


Fig. 3. Gas and material temperature of the pulse tube during compression and expansion. In the isothermal model, the gas temperature in the pulse tube during the compression $T_{\text{pt,c}}$ is assumed to be constant and slightly below the material temperature $T_{\text{m,pt}}$, whereas the gas temperature during the expansion $T_{\text{pt,e}}$ is assumed to be constant and slightly above the material temperature.

during the expansion stroke. The parameters ε_c and ε_e are in the range of $0 < \varepsilon_c < 1$ and $0 < \varepsilon_e < 1$, respectively.

At TDC, the pulse tube's gas temperature undergoes a thermal relaxation [11] from $T_{pt,c}$ to $T_{pt,e}$, and at BDC, a thermal relaxation from $T_{pt,e}$ to $T_{pt,c}$. Both relaxation processes occur due to shifting and mixing of the gas in the pulse accompanied by entropy production [29]. Since the relaxations take place at TDC and BDC, the gas volume of the entire engine is constant and the differential equations can be solved analytically [30].

Summarizing, the thermodynamic cycle is split into four phases. According to Fig. 4, an isothermal compression at $T_{pt,c}$ occurs between state 1 and state 2. A high pressure thermal relaxation from $T_{pt,c}$ to $T_{pt,e}$ takes place between state 2 and state 3 at constant minimum engine gas volume. An isothermal expansion at $T_{pt,e}$ occurs between state 3 and state 4 and a low pressure thermal relaxation from $T_{pt,e}$ to $T_{pt,c}$ takes place between state 4 and state 1 at constant maximum engine gas volume.

4.2. Boundary conditions

Between the engine components, boundary conditions for all transferred quantities need to be assigned.

The gas flowing between the components carries thermal energy and entropy. Both are related to the mass flux and the gas temperature. In Fig. 5, the temperatures of the gas crossing the control volume borders are designated. Gas leaving and entering the cylinder at its right side, respectively leaving and entering the cold heat exchanger at its left side has the temperature $T''_{cyl} = T'_{chx} = T_c$. Gas leaving and entering the hot heat exchanger at its right side, respectively leaving and entering the regenerator at its left side has the temperature $T''_{hhx} = T'_{reg} = T_h$. Gas leaving the cold heat exchanger at its right side, respectively entering the pulse tube at its left side has the temperature $T''_{chx} = T'_{pt} = T_c$. Gas leaving the hot heat exchanger at its left side, respectively entering the pulse tube at its right side has the temperature $T''_{pt} = T'_{hhx} = T_h$. During the compression stroke ($1 \rightarrow 2$), gas is leaving the pulse tube at its right side and entering the hot heat exchanger at its left side with the temperature $T''_{pt} = T'_{hhx} = T''_{pt,c}$, whereas during the high pressure thermal relaxation ($2 \rightarrow 3$), gas is leaving the pulse tube at its right side and entering the hot heat exchanger at its left side with the temperature $T''_{pt} = T'_{hhx} = T''_{pt,c} \dots T''_{pt,e}$. During the expansion stroke ($3 \rightarrow 4$), gas is leaving the pulse tube at its left side and entering the cold heat exchanger at its right side with the temperature $T''_{chx} = T'_{pt} = T'_{pt,e}$. During the low pressure thermal relaxation ($4 \rightarrow 1$), gas is entering the pulse tube at both sides.

Since all components except the pulse tube are isothermal throughout the entire cycle, the denotations ' and '' are only necessary to identify the boundary values at the pulse

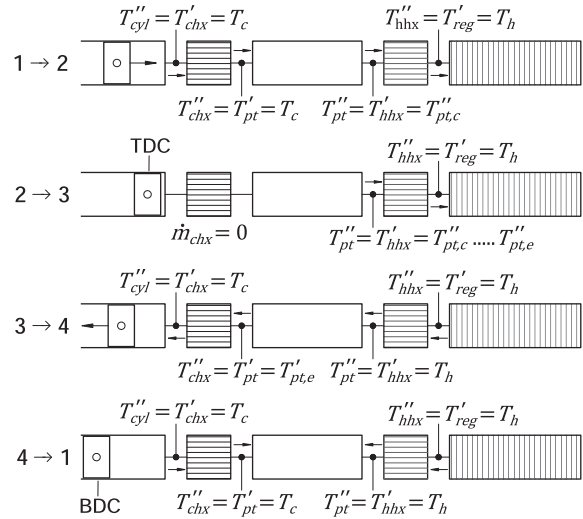


Fig. 5. Boundary gas temperatures of the engine components during the four phases of the cycle. T''_{cyl} : cylinder, T'_{chx} , T''_{chx} : cold heat exchanger, T'_{pt} , T''_{pt} : pulse tube, T'_{hhx} , T''_{hhx} : hot heat exchanger, T'_{reg} : regenerator. The arrows between the components symbolize the direction of the respective mass fluxes. The mass flux via the cold heat exchanger $\dot{m}_{chx}$ is zero during the high pressure thermal relaxation.

tube's cold ' and hot '' side. The respective pulse tube boundary temperatures T' and T'' are assumed to be

$$T' = \lambda' < T_{m,pt} > + (1 - \lambda') T_c$$

at its cold, and

$$T'' = \lambda'' < T_{m,pt} > + (1 - \lambda'') T_h$$

at its hot side.

During the compression stroke, the pulse tube's hot side exit temperature is $T''_{pt,c} = (1 - \varepsilon_c) T''$ and during the expansion stroke, the pulse tube's cold side exit temperature is $T'_{pt,e} = (1 + \varepsilon_e) T'$. The hot side exit temperature during the high pressure thermal relaxation is $T''_{pt,c} \dots T''_{pt,e} = (1 - \varepsilon_c) T'' \dots (1 + \varepsilon_e) T''$.

Gas is entering the pulse tube's cold side with the temperature T_c during the compression stroke and the pulse tube's hot side with the temperature T_h during the expansion stroke. Hence, the parameters λ' and λ'' can be deduced from the conditions $T'_{pt,c} = T_c$ and $T''_{pt,e} = T_h$ to

$$\lambda' = \frac{2\varepsilon_c}{(1 - \varepsilon_c)(\tau - 1)}$$

and

$$\lambda'' = \frac{2\varepsilon_e}{(1 + \varepsilon_e)(\tau - 1)}.$$

It follows

$$T'_{pt,e} = \frac{1 + \varepsilon_e}{1 - \varepsilon_c} T_c \quad \text{and} \quad T''_{pt,c} = \frac{1 - \varepsilon_c}{1 + \varepsilon_e} T_c \tau. \quad (10)$$

4.3. Calculation of the time-dependent pressure

Eq. (1) is used to calculate the pressure throughout the thermodynamic cycle of the pulse tube engine. For all considerations, the mass fluxes via the cold and hot heat exchanger are denoted by $\dot{m}_{chx}$ and $\dot{m}_{hhx}$. Not considering the heat exchanger gas volumes leads to $\dot{m}'_{chx} + \dot{m}''_{chx} = 0$ and $\dot{m}'_{hhx} + \dot{m}''_{hhx} = 0$, and thus

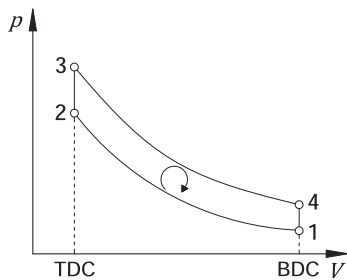


Fig. 4. pV -diagram of the idealized pulse tube engine cycle. TDC: piston's top dead center, BDC: piston's bottom dead center.

$\dot{m}'_{\text{chx}} = -\dot{m}''_{\text{chx}} = \dot{m}_{\text{chx}}$ and $\dot{m}'_{\text{hhx}} = -\dot{m}''_{\text{hhx}} = \dot{m}_{\text{hhx}}$, respectively. Left side mass fluxes $\dot{m}'_{\text{chx}}$ and $\dot{m}'_{\text{hhx}}$ are positive, and right side mass fluxes $\dot{m}''_{\text{chx}}$ and $\dot{m}''_{\text{hhx}}$ are negative.

With the definitions (6) and (7), Eq. (1) becomes

$$\frac{dp}{dt} = -\frac{R_s T_c}{V_{\text{cyl}}} \dot{m}_{\text{chx}} - \frac{p}{V_{\text{cyl}}} \frac{dV_{\text{cyl}}}{dt} \quad (11)$$

for the cylinder,

$$\frac{dp}{dt} = \frac{R_s T_{\text{pt}}}{V_d} r_c (r_d + 1) (\dot{m}_{\text{chx}} - \dot{m}_{\text{hhx}}) \quad (12)$$

for the pulse tube during the isothermal compression and the isothermal expansion,

$$\frac{dp}{dt} = \frac{R_s T_{\text{pt}}}{V_d} r_c (r_d + 1) (\dot{m}_{\text{chx}} - \dot{m}_{\text{hhx}}) + \frac{p}{T_{\text{pt}}} \frac{dT_{\text{pt}}}{dt} \quad (13)$$

for the pulse tube during the thermal relaxations, and

$$\frac{dp}{dt} = \frac{R_s T_{\text{reg}}}{V_d} \frac{r_c (r_d + 1)}{r_d} \dot{m}_{\text{hhx}} \quad (14)$$

for the regenerator.

The cylinder gas volume at BDC is $V_{\text{cyl}} = V_d$ and at TDC $V_{\text{cyl}} = 0$. The resulting mass fluxes can be calculated from Eqs. (11) or (12) and Eq. (14). The mass flux via the cold heat exchanger is during the isothermal compression and the isothermal expansion

$$\dot{m}_{\text{chx,c/e}} = \frac{V_d}{R_s r_c (r_d + 1)} \left(\frac{r_d}{T_{\text{reg}}} + \frac{1}{T_{\text{pt}}} \right) \frac{dp}{dt}, \quad (15)$$

during the high pressure thermal relaxation

$$\dot{m}_{\text{chx,hp}} = 0, \quad (16)$$

and during the low pressure thermal relaxation

$$\dot{m}_{\text{chx,lp}} = -\frac{V_d}{R_s T_c} \frac{dp}{dt}. \quad (17)$$

Throughout all phases, the mass flux via the hot heat exchanger is given by

$$\dot{m}_{\text{hhx}} = \frac{V_d}{R_s r_c (r_d + 1)} \frac{r_d}{T_{\text{reg}}} \frac{dp}{dt}. \quad (18)$$

The pressure during the four engine phases is calculated as follows:

Isothermal compression from state 1 to state 2

Inserting Eq. (15) in Eq. (11) leads to

$$\frac{1}{p} \frac{dp}{dt} = -\frac{1}{V_{\text{cyl}} + C_c} \frac{dV_{\text{cyl}}}{dt}, \quad (19)$$

with the abbreviation

$$C_c := \frac{V_d}{r_c (r_d + 1)} \left(r_d \frac{T_c}{T_{\text{reg}}} + \frac{T_c}{T_{\text{pt,c}}} \right). \quad (20)$$

Integrating Eq. (19) with the initial conditions $V_{\text{cyl}}(t_1) = V_1 = V_d$ and $p(t_1) = p_1 = p_{\text{min}}$ leads to the pressure during the compression stroke

$$p_c(t) = p_1 \frac{V_d + C_c}{V_{\text{cyl}}(t) + C_c}, \quad (21)$$

which reaches in state 2

$$p_2 = p_1 \frac{V_d + C_c}{C_c}. \quad (22)$$

Thermal relaxation from state 2 to state 3

Inserting Eqs. (16) and (18) in Eq. (13), together with the abbreviation

$$C_{\text{hp}} := \frac{r_d}{T_{\text{reg}}}, \quad (23)$$

results in

$$\frac{1}{p} \frac{dp}{dt} = -\frac{1}{T_{\text{pt}} + C_{\text{hp}} T_{\text{pt}}^2} \frac{dT_{\text{pt}}}{dt}. \quad (24)$$

Integrating Eq. (24) from state 2 to state 3 leads to the pressure

$$p_3 = p_2 \frac{(C_{\text{hp}} T_{\text{pt,c}} + 1) T_{\text{pt,e}}}{(C_{\text{hp}} T_{\text{pt,e}} + 1) T_{\text{pt,c}}}. \quad (25)$$

Isothermal expansion from state 3 to state 4

Inserting Eq. (15) in Eq. (11) leads to

$$\frac{1}{p} \frac{dp}{dt} = -\frac{1}{V_{\text{cyl}} + C_e} \frac{dV_{\text{cyl}}}{dt}, \quad (26)$$

with the abbreviation

$$C_e := \frac{V_d}{r_c (r_d + 1)} \left(r_d \frac{T_c}{T_{\text{reg}}} + \frac{T_c}{T_{\text{pt,e}}} \right). \quad (27)$$

Integrating Eq. (26) with the initial conditions $V_{\text{cyl}}(t_3) = V_3 = 0$ and $p(t_3) = p_3 = p_{\text{max}}$ leads to the pressure during the expansion stroke

$$p_e(t) = p_3 \frac{C_e}{V_{\text{cyl}}(t) + C_e}, \quad (28)$$

which reaches in state 4

$$p_4 = p_3 \frac{C_e}{V_d + C_e}. \quad (29)$$

Thermal relaxation from state 4 to state 1

Inserting Eqs. (17) and (18) in Eq. (13), together with the abbreviation

$$C_{\text{lp}} := \frac{r_c (r_d + 1)}{T_c} + \frac{r_d}{T_{\text{reg}}}, \quad (30)$$

results in

$$\frac{1}{p} \frac{dp}{dt} = -\frac{1}{T_{\text{pt}} + C_{\text{lp}} T_{\text{pt}}^2} \frac{dT_{\text{pt}}}{dt}. \quad (31)$$

Integrating Eq. (31) from state 4 to state 1 leads to the pressure

$$p_1^* = p_4 \frac{(C_{lp} T_{pt,e} + 1) T_{pt,c}}{(C_{lp} T_{pt,c} + 1) T_{pt,e}}. \quad (32)$$

Closing the cycle through the combination of Eqs. (22), (25), (29), and (32) demands

$$\frac{p_1^*}{p_1} = \frac{V_d + C_c}{C_c} \frac{(C_{hp} T_{pt,c} + 1) T_{pt,e}}{(C_{hp} T_{pt,e} + 1) T_{pt,c}} \frac{C_e}{V_d + C_e} \frac{(C_{lp} T_{pt,e} + 1) T_{pt,c}}{(C_{lp} T_{pt,c} + 1) T_{pt,e}} \stackrel{!}{=} 1, \quad (33)$$

which is fulfilled for all values of ε_c and ε_e . Without loss of generality, the ratio between ε_c and ε_e is defined as

$$\tau_0 := \frac{\varepsilon_c}{\varepsilon_e}. \quad (34)$$

With $\varepsilon := \varepsilon_e$ results $\varepsilon_c = \tau_0 \varepsilon$. The value of ε can be obtained from the mean gas temperature in the pulse tube during compression

$$T_{pt,c} = \frac{1}{2} (T'_{pt,c} + T''_{pt,c}) \quad (35)$$

or during expansion

$$T_{pt,e} = \frac{1}{2} (T'_{pt,e} + T''_{pt,e}), \quad (36)$$

respectively. Eqs. (35) and (36) lead to the same result for

$$\varepsilon = \frac{\tau/\tau_0 - 1}{\tau + 1}. \quad (37)$$

4.4. Calculation of the pV -work and transferred heat

The pV -work W_{pV} is calculated through integration of the isothermal compression power $\dot{W}_c$ and the isothermal expansion power $\dot{W}_e$ according to

$$W_{pV} = \int_1^2 \dot{W}_c dt + \int_3^4 \dot{W}_e dt = - \int_{V_d}^0 p_c dV_{cyl} - \int_0^{V_d} p_e dV_{cyl}. \quad (38)$$

Eq. (38) results with Eqs. (21) and (28) in

$$W_{pV} = p_{\min} (V_d + C_c) \ln \left(\frac{V_d}{C_c} + 1 \right) - p_{\max} C_e \ln \left(\frac{V_d}{C_e} + 1 \right). \quad (39)$$

The heat fluxes $\dot{Q}_{cyl}$, $\dot{Q}_{pt}$, and $\dot{Q}_{reg}$ to the gas in the respective components are calculated using Eq. (2), which, for the cylinder becomes

$$0 = c_v T_c \dot{m}_{chx} + \dot{Q}_{cyl} - p \frac{dV_{cyl}}{dt} - \dot{m}_{chx} c_p T_c, \quad (40)$$

for the pulse tube during the compression and expansion stroke

$$0 = c_v T_{pt} (\dot{m}_{hhx} - \dot{m}_{chx}) + \dot{Q}_{pt} + \dot{m}_{chx} c_p T'' - \dot{m}_{hhx} c_p T', \quad (41)$$

for the pulse tube during the thermal relaxations with the definitions (6) and (7)

$$\frac{c_v p V_d}{R_s r_c (r_d + 1)} \frac{dT_{pt}}{dt} = c_v T (\dot{m}_{hhx} - \dot{m}_{chx}) + \dot{Q}_{pt} + \dot{m}_{chx} c_p T'' - \dot{m}_{hhx} c_p T', \quad (42)$$

and for the regenerator

$$0 = -c_v T_{reg} \dot{m}_{hhx} + \dot{Q}_{reg} + \dot{m}_{hhx} c_p T_h. \quad (43)$$

Using Eq. (2), the heat fluxes in the heat exchangers are calculated for the cold heat exchanger with

$$\dot{Q}_{chx} = \dot{m}_{chx} c_p (T'' - T_c), \quad (44)$$

and for the hot heat exchanger with

$$\dot{Q}_{hhx} = \dot{m}_{hhx} c_p (T_h - T'), \quad (45)$$

respectively.

Solving Eq. (40)–(45) for the four phases and integrating the heat fluxes over the entire engine cycle leads to the amounts of heat transferred to the working gas or removed from the working gas in the components per cycle according to

$$Q_{cyl/chx/pt/hhx} = \oint \dot{Q}_{cyl/chx/pt/hhx} dt. \quad (46)$$

The amount of heat transferred per cycle in the regenerator is

$$Q_{reg} = \oint \dot{Q}_{reg} dt = 0. \quad (47)$$

4.5. Calculation of the entropy production

To understand the power and efficiency limitations of the pulse tube engine, the entropy production rate is calculated using Eq. (3).

For the cylinder follows

$$\begin{aligned} \frac{dS_{irr,cyl}}{dt} &= -\dot{m}_{chx} \left(c_v \left(\ln \frac{T_c}{T^0} - 1 \right) + R_s \ln \frac{V_{cyl}}{V^0} \right) \\ &\quad - \dot{m}_{chx} \left(c_p \left(1 - \ln \frac{T_c}{T^0} \right) + R_s \ln \frac{p}{p^0} \right) \\ &= \dot{m}_{chx} R_s \left(\ln \frac{T_c}{T^0} - \ln \frac{V_{cyl}}{V^0} - \ln \frac{p}{p^0} - 1 \right), \end{aligned} \quad (48)$$

which, together with the thermal state equation of a perfect gas, results in

$$\begin{aligned} \frac{dS_{irr,cyl}}{dt} &= \dot{m}_{chx} R_s \left(\ln \left(\frac{T_c}{p V_{cyl}} \frac{p^0 V^0}{T^0} \right) - 1 \right) \\ &= -\dot{m}_{chx} R_s \left(\ln \frac{m_{cyl}}{m^0} + 1 \right), \end{aligned}$$

where m_{cyl} represents the gas mass content of the cylinder and m^0 represents a constant. Cycle integration, together with $\dot{m}_{chx} = -dm_{cyl}/dt$ and

$$\oint dm = 0, \quad (49)$$

leads to

$$S_{irr,cyl} = \oint dS_{irr,cyl} = R_s \oint \left(\ln \frac{m_{cyl}}{m^0} + 1 \right) dm_{cyl} = 0. \quad (50)$$

The entropy production rate in the cold heat exchanger is

$$\begin{aligned} \frac{dS_{\text{irr, chx}}}{dt} &= \dot{m}_{\text{chx}} c_p \left(1 - \ln \frac{T_c}{T_0} \right) - \dot{m}_{\text{chx}} c_p \left(\frac{T'_{\text{chx}}}{T_c} - \ln \frac{T''_{\text{chx}}}{T_0} \right) \\ &= \dot{m}_{\text{chx}} c_p \left(1 - \frac{T'_{\text{chx}}}{T_c} + \ln \frac{T''_{\text{chx}}}{T_c} \right). \end{aligned} \quad (51)$$

Cycle integration, together with Eqs. (10), (27), (29), and (34), as well as $\varepsilon = \varepsilon_e$ and $\varepsilon_c = \tau_0 \varepsilon$, leads to

$$S_{\text{irr, chx}} = \frac{c_p p_{\text{max}}}{R_s T_c} \frac{V_d C_e}{V_d + C_e} \left(\frac{(\tau_0 + 1)\varepsilon}{1 - \tau_0 \varepsilon} - \ln \frac{1 + \varepsilon}{1 - \tau_0 \varepsilon} \right). \quad (52)$$

The entropy production rate in the pulse tube is given by

$$\begin{aligned} \frac{dS_{\text{irr, pt}}}{dt} &= (\dot{m}_{\text{chx}} - \dot{m}_{\text{hhx}}) \left(c_v \left(\ln \frac{T_{\text{pt}}}{T_0} - 1 \right) + R_s \ln \frac{V_{\text{pt}}}{V_0} \right) \\ &\quad + \dot{m}_{\text{chx}} \left(c_p \left(\frac{T'_{\text{pt}}}{T_{\text{pt}}} - \ln \frac{T'_{\text{pt}}}{T_0} \right) + R_s \ln \frac{p}{p_0} \right) \\ &\quad - \dot{m}_{\text{hhx}} \left(c_p \left(\frac{T''_{\text{pt}}}{T_{\text{pt}}} - \ln \frac{T''_{\text{pt}}}{T_0} \right) + R_s \ln \frac{p}{p_0} \right) \\ &= (\dot{m}_{\text{chx}} - \dot{m}_{\text{hhx}}) \left(c_v \left(\ln \frac{T_{\text{pt}}}{T_0} - 1 \right) + R_s \left(\ln \frac{V_{\text{pt}}}{V_0} + \ln \frac{p}{p_0} \right) \right) \\ &\quad + \dot{m}_{\text{chx}} c_p \left(\frac{T'_{\text{pt}}}{T_{\text{pt}}} - \ln \frac{T'_{\text{pt}}}{T_0} \right) - \dot{m}_{\text{hhx}} c_p \left(\frac{T''_{\text{pt}}}{T_{\text{pt}}} - \ln \frac{T''_{\text{pt}}}{T_0} \right), \end{aligned} \quad (53)$$

which, together with $\dot{m}_{\text{chx}} - \dot{m}_{\text{hhx}} = dm_{\text{pt}}/dt$, the thermal state equation of a perfect gas, and $R_s = c_p - c_v$, becomes

$$\begin{aligned} \frac{dS_{\text{irr, pt}}}{dt} &= (\dot{m}_{\text{chx}} - \dot{m}_{\text{hhx}}) c_p \ln \frac{T_{\text{pt}}}{T_0} + \frac{dm_{\text{pt}}}{dt} \left(R_s \ln \frac{m_{\text{pt}}}{m^0} - c_v \right) \\ &\quad + \dot{m}_{\text{chx}} c_p \left(\frac{T'_{\text{pt}}}{T_{\text{pt}}} - \ln \frac{T'_{\text{pt}}}{T_0} \right) - \dot{m}_{\text{hhx}} c_p \left(\frac{T''_{\text{pt}}}{T_{\text{pt}}} - \ln \frac{T''_{\text{pt}}}{T_0} \right), \end{aligned}$$

where the gas mass content of the pulse tube is m_{pt} and m^0 is a constant. Cycle integration, together with Eq. (49), leads to

$$\begin{aligned} S_{\text{irr, pt}} &= c_p \oint (\dot{m}_{\text{chx}} - \dot{m}_{\text{hhx}}) \ln \frac{T_{\text{pt}}}{T_0} dt + c_p \oint \left(\dot{m}_{\text{chx}} \left(\frac{T'_{\text{pt}}}{T_{\text{pt}}} - \ln \frac{T'_{\text{pt}}}{T_0} \right) \right. \\ &\quad \left. - \dot{m}_{\text{hhx}} \left(\frac{T''_{\text{pt}}}{T_{\text{pt}}} - \ln \frac{T''_{\text{pt}}}{T_0} \right) \right) dt. \end{aligned} \quad (54)$$

The entropy production rate in the hot heat exchanger is

$$\begin{aligned} \frac{dS_{\text{irr, hhx}}}{dt} &= \dot{m}_{\text{hhx}} c_p \left(\frac{T'_{\text{hhx}}}{T_h} - \ln \frac{T'_{\text{hhx}}}{T_0} \right) - \dot{m}_{\text{hhx}} c_p \left(1 - \ln \frac{T_h}{T_0} \right) \\ &= \dot{m}_{\text{hhx}} c_p \left(\frac{T'_{\text{hhx}}}{T_h} - 1 - \ln \frac{T'_{\text{hhx}}}{T_h} \right). \end{aligned} \quad (55)$$

Cycle integration, together with Eqs. (10), (20), (22), and (34), as well as $\varepsilon = \varepsilon_e$ and $\varepsilon_c = \tau_0 \varepsilon$, leads to

$$\begin{aligned} S_{\text{irr, hhx}} &= \frac{c_p p_{\text{min}}}{R_s T_c} \frac{V_p r_d (1 - \tau_0 \varepsilon)}{r_d (1 - \tau_0 \varepsilon) + 1} \left(\ln \frac{1 + \varepsilon}{1 - \tau_0 \varepsilon} - \frac{(\tau_0 + 1)\varepsilon}{1 + \varepsilon} \right) \\ &\quad + c_p \int_2^3 \dot{m}_{\text{hhx}} \left(\frac{T'_{\text{hhx}}}{T_h} - 1 - \ln \frac{T'_{\text{hhx}}}{T_h} \right) dt. \end{aligned} \quad (56)$$

For the entropy production rate in the regenerator follows

$$\begin{aligned} \frac{dS_{\text{irr, reg}}}{dt} &= \dot{m}_{\text{hhx}} \left(c_v \left(\ln \frac{T_{\text{reg}}}{T_0} - 1 \right) + R_s \ln \frac{V_{\text{reg}}}{V_0} \right) + \dot{m}_{\text{hhx}} \left(c_p \left(\frac{T_h}{T_{\text{reg}}} \right. \right. \\ &\quad \left. \left. - \ln \frac{T_h}{T_0} \right) + R_s \ln \frac{p}{p_0} \right), \end{aligned} \quad (57)$$

in which all terms except of p and $\dot{m}_{\text{hhx}}$ are constant. Thus, its cycle integration leads to

$$\begin{aligned} S_{\text{irr, reg}} &= \oint dS_{\text{irr, cyl}} = \left(c_v \left(\ln \frac{T_{\text{reg}}}{T_0} - 1 \right) + c_v \left(\frac{T_h}{T_{\text{reg}}} - \ln \frac{T_h}{T_0} \right) \right. \\ &\quad \left. + R_s \ln \frac{V_{\text{reg}}}{V_0} \right) \oint d\dot{m}_{\text{hhx}} \\ &\quad + R_s \oint \dot{m}_{\text{hhx}} \ln \frac{p}{p_0} dt \stackrel{(49)}{=} R_s \oint \dot{m}_{\text{hhx}} \ln \frac{p}{p_0} dt. \end{aligned} \quad (58)$$

With Eq. (18) Eq. (58) becomes

$$S_{\text{irr, reg}} = \frac{V_d}{(r_c - 1)(r_d + 1)} \frac{r_d}{T_{\text{reg}}} \oint \ln \frac{p}{p_0} dp \stackrel{(33)}{=} 0. \quad (59)$$

5. Results

A virtual pulse tube engine having the characteristics $r_c = r_d = 1$, $p_{\text{min}} V_d = 1$ J, and $T_c = 300$ K has been simulated. It uses the perfect gas Helium with $\kappa = 1.66$ as working fluid. The minimal pressure is $p_{\text{min}} = 1$ bar and the displacement volume is $V_d = 10$ cm³. The thermodynamic cycle was calculated for temperature ratios between 1 ($T_h = 300$ K) and 5 ($T_h = 1500$ K).

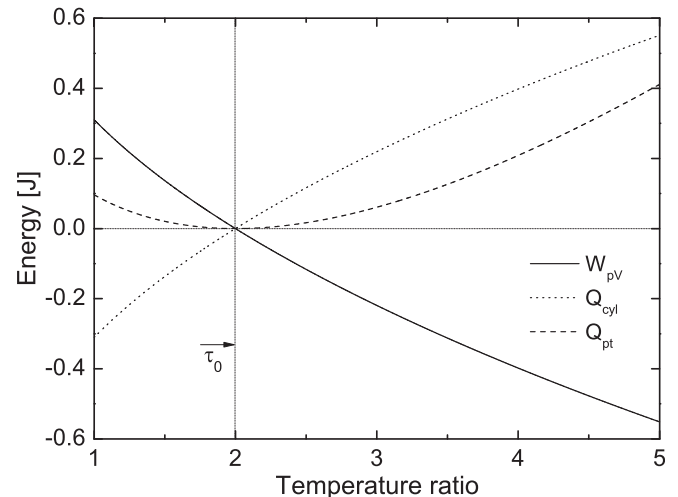


Fig. 6. Analytically calculated pV -work and amounts of heat transferred per cycle in the cylinder and the pulse tube as functions of the temperature ratio.

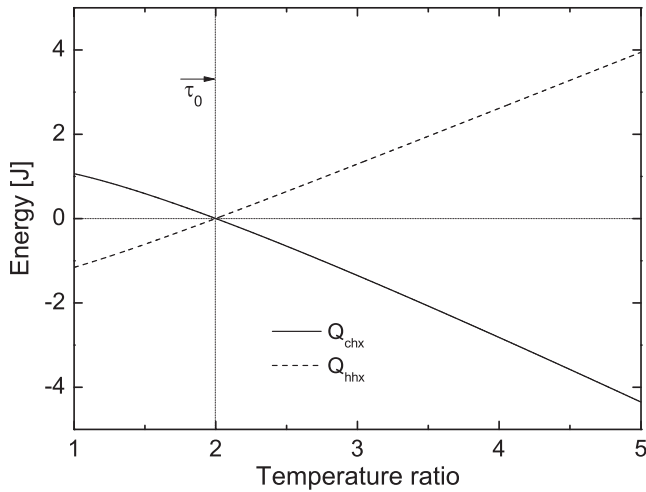


Fig. 7. Analytically calculated amounts of heat transferred per cycle in the cold and hot heat exchanger as functions of the temperature ratio.

5.1. Calculated pV -work, transferred heat, thermal efficiency, and entropy production

Figs. 6 and 7 show the pV -work, calculated with Eq. (39), and the amounts of heat transferred per cycle in the engine components, calculated with Eq. (46). All energies are zero at $\tau = \tau_0 = 2$. For $\tau < \tau_0$, the pV -work is positive and work has to be delivered to the engine, which is thus not acting as a prime mover. In this case heat is added to the working gas in the cold heat exchanger and rejected from it in the hot heat exchanger. The engine operates as a heat pump. For $\tau > \tau_0$, the pV -work is negative and the engine operates as a prime mover. Heat is added to the working gas in the hot heat exchanger and rejected from it in the cold heat exchanger. The amount of heat transferred per cycle in the pulse tube is much smaller than in the other components. According to Fig. 6, the heat transferred per cycle in the pulse tube has an extreme value at $\tau = \tau_0$. Thus,

$$\left. \frac{dQ_{pt}}{d\tau} \right|_{\tau=\tau_0} = -\frac{\kappa p_{\min} V_d (r_d (\tau_0 - 1) - 1)}{(\kappa - 1)(r_d + 1)\tau_0} \stackrel{!}{=} 0. \quad (60)$$

With Eq. (60) the minimal temperature ratio above which the engine provides a work output results in

$$\tau_0 = \frac{r_d + 1}{r_d}. \quad (61)$$

The second derivative of Q_{pt} in τ at $\tau = \tau_0$ is

$$\left. \frac{d^2 Q_{pt}}{d\tau^2} \right|_{\tau=\tau_0} = \frac{\kappa p_{\min} V_d r_d^2}{r_c (\kappa - 1)} \frac{4r_d^2 (r_c - 1) + 2r_c r_d (r_c + 2) + r_c^2}{(r_d + 1)^2 (r_d (2r_d + 3) + 1) (2r_d (r_c + 1) + r_c)}. \quad (62)$$

The sign of Eq. (62) is governed by

$$\sigma = 4r_d^2 (r_c - 1) + 2r_c r_d (r_c + 2) + r_c^2.$$

If $r_c < 1$ and

$$r_c < 2r_d \frac{\sqrt{r_d^2 + 4r_d + 2} - (r_d + 1)}{2r_d + 1},$$

σ is smaller than zero and thus $d^2 Q_{pt}/d\tau^2|_{\tau=\tau_0} < 0$. In this case, at $\tau = \tau_0$, a local maximum of Q_{pt} is located and integrated over one cycle heat is removed from the working gas in the pulse tube.

If $r_c < 1$ and

$$r_c = 2r_d \frac{\sqrt{r_d^2 + 4r_d + 2} - (r_d + 1)}{2r_d + 1},$$

σ is equal to zero and thus $d^2 Q_{pt}/d\tau^2|_{\tau=\tau_0} = 0$. In this case, integrated over the cycle, no heat is exchanged between the working gas and the pulse tube material. In all other cases, integrated over the cycle, heat is added to the working gas in the pulse tube. This situation allows for an addition of heat to the working gas at temperatures lower than T_h , enabling the engine to convert heat energy from sources with decreasing temperature. Only this case is of practical interest.

The thermal efficiency is calculated with

$$\eta_{th} = \frac{|W_{pV}|}{Q_{in}}, \quad (63)$$

where Q_{in} represents the heat input. According to Figs. 6 and 7, for $\tau = \tau_0$, heat is added to the working gas in the cylinder at the temperature T_c , in the hot heat exchanger at the temperature T_h , and for $\sigma > 0$ in the pulse tube at temperatures between T_c and T_h . The heat added to the working gas in the cylinder, in general, is provided from the surroundings and does not have to be provided by the heat source. Taking into account only the heat delivered by the hot heat exchanger at the temperature T_h , the heat input is calculated with $Q_{in} = Q_{hhx}$. Considering in the case of $\sigma > 0$ also the heat added to the working gas in the pulse tube at temperatures between T_c and T_h , the heat input is calculated with $Q_{in} = Q_{hhx} + Q_{pt}$. In both cases, for $\tau \rightarrow \tau_0$, Eq. (63) becomes finite, equaling to

$$\eta_{th} = \frac{\kappa - 1}{\kappa(r_d + 1)}, \quad (64)$$

which is only possible if $\tau_0 > 1$.

In Fig. 8, the thermal efficiency of the pulse tube engine, calculated with Eq. (63), is shown for temperature ratios between $\tau = \tau_0$ and $\tau = 5$. At $\tau = \tau_0$, the efficiency is highest, given by Eq. (64), and decreases monotonously for $\tau > \tau_0$. The efficiency calculated

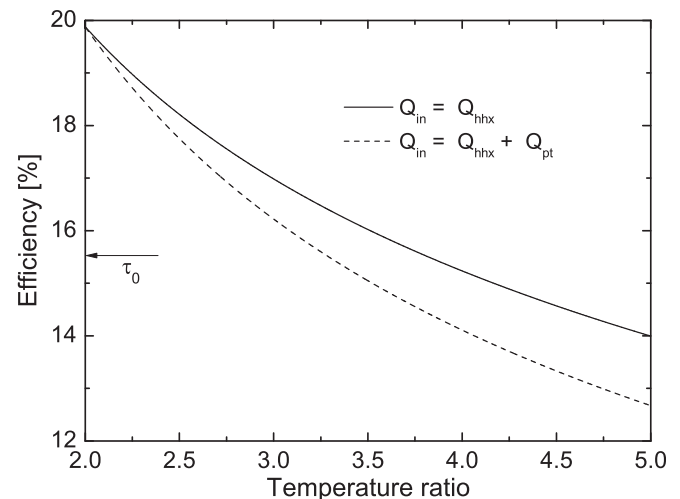


Fig. 8. Analytically calculated thermal efficiency of the pulse tube engine as a function of the temperature ratio.

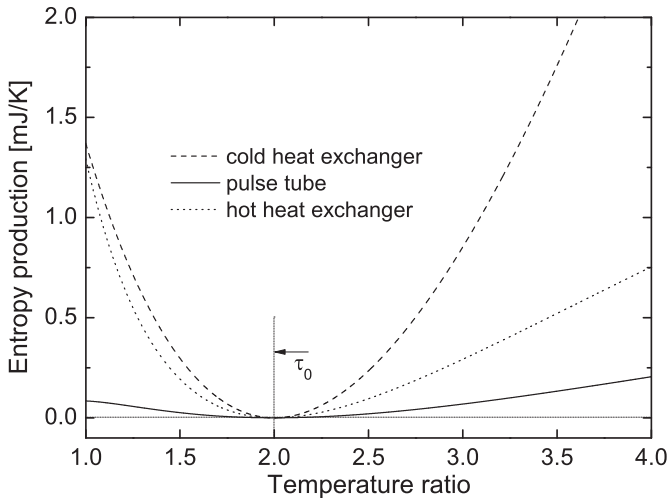


Fig. 9. Analytically calculated entropy production per cycle in the cold and hot heat exchanger as well as in the pulse tube as functions of the temperature ratio.

with $Q_{in} = Q_{hhx}$ is higher than the efficiency calculated with $Q_{in} = Q_{hhx} + Q_{pt}$, but in the case of $Q_{in} = Q_{hhx} + Q_{pt}$, heat is added to the working gas at temperatures between T_c and T_h in the pulse tube.

The decreasing nature of the thermal efficiency for $\tau > \tau_0$ is based on the working principle of the pulse tube engine. This becomes obvious regarding Fig. 9, which shows the entropy production in the cold heat exchanger, the pulse tube, and the hot heat exchanger, calculated with Eqs. (52), (54), and (56). For $\tau \neq \tau_0$, entropy is produced in all components, which is caused by mixing of gas at different temperatures. Since a pV -work output is only provided for $\tau > \tau_0$, the pulse tube engine is intrinsically irreversible. This was also concluded from experimental investigations of the pulse tube engine by Yoshida et al. [13].

5.2. Influence of operating and design parameters on the engine cycle

The time-dependent pressure of the engine during compression and expansion can be calculated with Eqs. (21) and (28). The results are used to analyze the engine cycle.

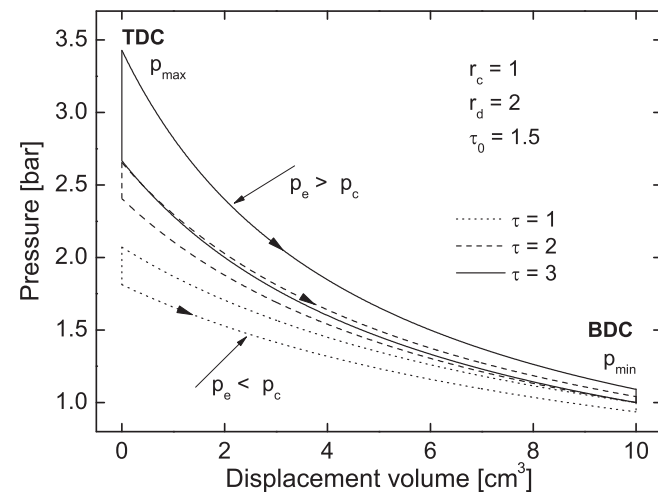


Fig. 10. Analytically calculated pV -diagram of the virtual pulse tube engine for $r_c = 1$, $r_d = 2$ and temperature ratios of 1, 2, and 3.

The influence of the temperature ratio on the thermodynamic cycle was investigated. Fig. 10 shows the resulting pV -diagram of the virtual pulse tube engine for $r_c = 1$, $r_d = 2$ and temperature ratios of 1, 2, and 3. The minimal temperature ratio for engine operation is $\tau_0 = 1.5$ ($T_h = 450$ K). The slopes of the isothermal compression and expansion increase with an increase of τ . For $\tau = 1$, the expansion stroke is on a lower pressure level than the compression stroke. In this case, pV -work has to be delivered to the engine. For $\tau = 2$ and $\tau = 3$, the expansion stroke is on a higher pressure level than the compression stroke and the engine provides a pV -work output. The intensity of the isochoric pressure rise at TDC is highly dependent on τ . In the case of $\tau = 1$, the gas in the pulse tube is cooled at the end of the compression stroke, since its compression end temperature is higher than the pulse tube's wall material temperature. For higher temperature ratios, the compression end temperature is lower than the pulse tube's wall material temperature resulting in a heat addition to the working gas at TDC. The same happens at BDC, but with an opposite sign of the heat flux in the pulse tube. If the temperature ratio is too small, the working gas expands below the pulse tube's wall material temperature and takes on heat at BDC. Having higher temperature ratios, the gas temperature in the pulse tube does not fall under the pulse tube's wall material temperature and heat is rejected at BDC.

Next, the influence of the compression ratio on the engine cycle is analyzed. In Fig. 11, the pV -diagram of the virtual pulse tube engine for $r_d = 2$, $\tau = 2$ and compression ratios of 0.001, 1, and 2 is shown. As expected from the theory, the pressure rise during compression increases for higher values of r_c . For all compression ratios, the isothermal expansion is on a higher pressure level than the isothermal compression. Thus, the engine provides a work output for all values of $r_c > 0$. The surface area enclosed by the pV -curve is comparable for all compression ratios. For $r_c \rightarrow 0$, the temperature change during compression and expansion becomes zero. According to Eq. (54), the entropy production in the pulse tube becomes singular, and thus, the energy dissipation becomes infinite. This hampers an operation of the engine for $r_c \rightarrow 0$. The calculated work output for $r_c \rightarrow 0$ is only a theoretical possible value for $Q_{in} \rightarrow \infty$.

According to Eq. (39), the pV -work is only of minor order dependent on r_c . To show this, the pV -work is plotted in Fig. 12 as a function of the compression ratio, for $r_d = 2$ and temperature ratios of 2, 3, 4, and 5. The values are normalized to the value for $r_c = 0$. The dependency of the pV -work on r_c is evident. For small

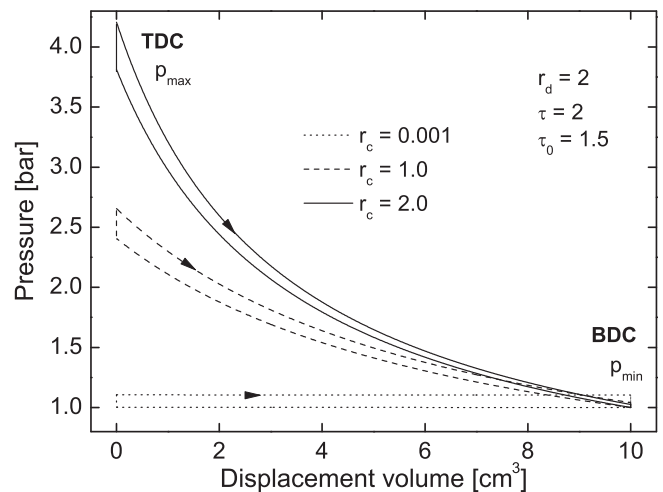


Fig. 11. Analytically calculated pV -diagram of the virtual pulse tube engine for $\tau = 2$, $r_d = 2$ and compression ratios of 0.001, 1, and 2.

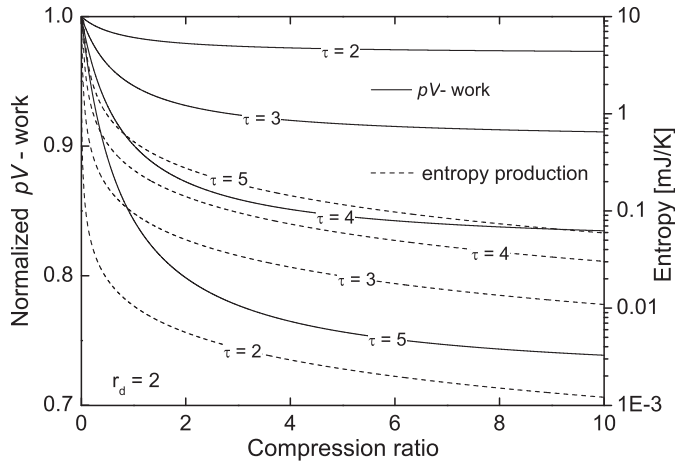


Fig. 12. Analytically calculated normalized pV -work as a function of the compression ratio and entropy production in the pulse tube as a function of the compression ratio of the virtual pulse tube engine having a distribution ratio of 2 and temperature ratios of 2, 3, 4, and 5.

temperature ratios, the influence of r_c is insignificant. Nevertheless, an increase of r_c slightly reduces the pV -work of the ideal pulse tube engine. The influence of the compression ratio on the pV -work increases with the temperature ratio. On the other hand, according to Eq. (54), an increase of r_c decreases the entropy production in the pulse tube. This is also shown in Fig. 12. Thus, the power output of the real pulse tube engine should have an optimal compression ratio at which the power output is best. This peak value should be located in the region of high $dS_{irr,pt}/d\tau$ around $r_c = 1$, which was proven numerically in Ref. [23]. There, the used compression ratio r_c^* is defined as $r_c^* = r_c + 1$.

The thermal efficiency, calculated with Eq. (63), is shown in Fig. 13. With increasing compression ratios up to $r_c = 1$, the thermal efficiency increases rapidly and remains constant for higher compression ratios. This strengthens the result of a peak pV -work around $r_c = 1$.

Finally, the distribution ratio is studied with respect to its influence on the engine cycle. Fig. 14 shows the pV -diagram of the virtual pulse tube engine for $r_c = 1$, $\tau = 2$ and distribution ratios of 0.5, 1.2, and 3. For $r_d = 0.5$, the engine does not provide a work output. Increasing the distribution ratio enables the cycle to

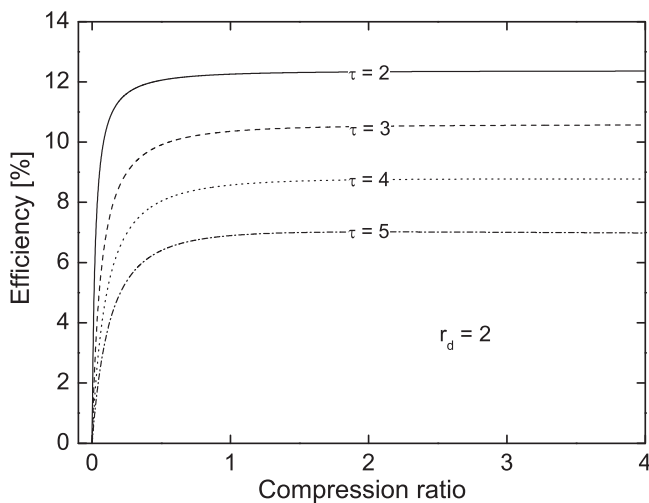


Fig. 13. Analytically calculated thermal efficiency as a function of the compression ratio of the virtual pulse tube engine corresponding to the conditions of Fig. 12.

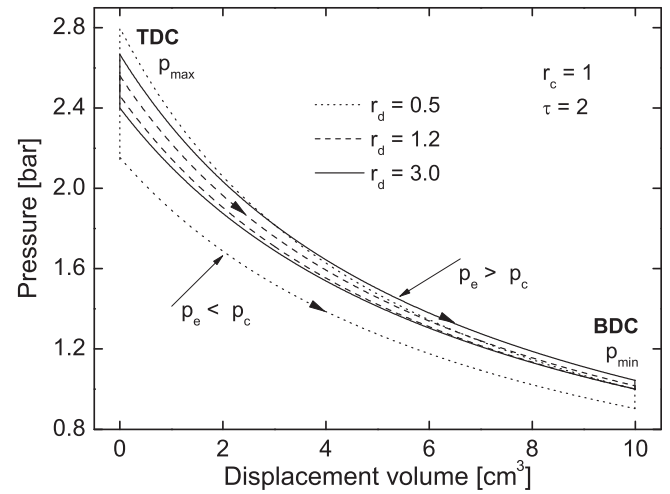


Fig. 14. Analytically calculated pV -diagram of the virtual pulse tube engine for $\tau = 2$, $r_c = 1$ and distribution ratios of 0.5, 1.2, and 3.

convert heat into work. An increase of the distribution ratio increases the regenerator volume and decreases the pulse tube volume. This directly influences the thermodynamic asymmetry caused by the arrangement of pulse tube and regenerator. A higher value of r_d increases the mass flux passing the pulse tube and the hot heat exchanger, but decreases the mass content experiencing a thermal relaxation in the pulse tube. Thus, an optimal value of r_d , maximizing the pV -work, exists. The linear component of the first derivative of the Eq. (39) in r_d is

$$\frac{dW_{pV,I}}{dr_d} = p_{\min} V_d \frac{r_d(\tau - 1) - (\tau + 1)}{(r_d + 1)^3}, \quad (65)$$

which becomes zero for

$$r_{d,opt} \approx \frac{\tau + 1}{\tau - 1}. \quad (66)$$

For different temperature ratios, the pV -work is calculated and shown in Fig. 15. The strong dependency of the pV -work on the distribution ratio is evident. The influence of the regenerator on the cycle becomes obvious. The lower the available temperature ratio,

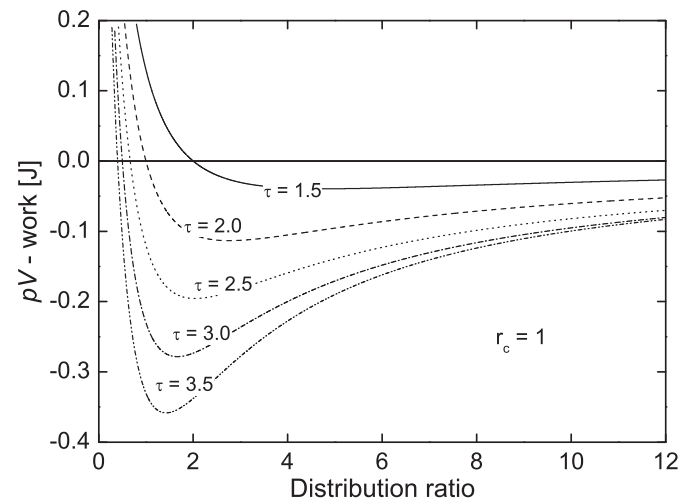


Fig. 15. Analytically calculated pV -work as a function of the distribution ratio of the virtual pulse tube engine having a compression ratio of 1 and temperature ratios of 1.5, 2, 2.5, 3, and 3.5.

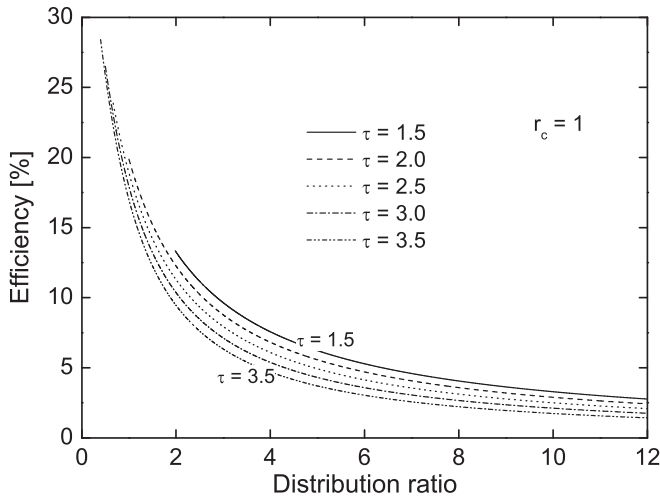


Fig. 16. Analytically calculated thermal efficiency as a function of the distribution ratio of the virtual pulse tube engine corresponding to the conditions of Fig. 15.

the higher the value of r_d and thus, the higher the necessary regenerator gas volume to enable a pV -work output. As expected, exceeding the optimal distribution ratio leads to a decrease of the pV -work output. A similar result was obtained numerically in Ref. [23]. Since according to Eq. (66) the peak value is a function of the temperature ratio, a careful design of the pulse tube engine with respect to the distribution ratio is necessary. The found relation in Eq. (66) delivers an approximation of the optimal distribution ratio. In contrast, the thermal efficiency, calculated with Eq. (63) and shown in Fig. 16, does not have a peak value at any distribution ratio. To increase the thermal efficiency, the distribution ratio has to be small. According to Eq. (61), the minimal temperature ratio τ_0 above which the engine provides a work output is a function of r_d . Thus, the lowest possible value of r_d is limited by the available temperature ratio.

5.3. Comparison of the theoretical results with experimental data

In Refs. [4,24] a working pulse tube engine was presented. The experimental test setup is used to prove the predictions of the analytical model.

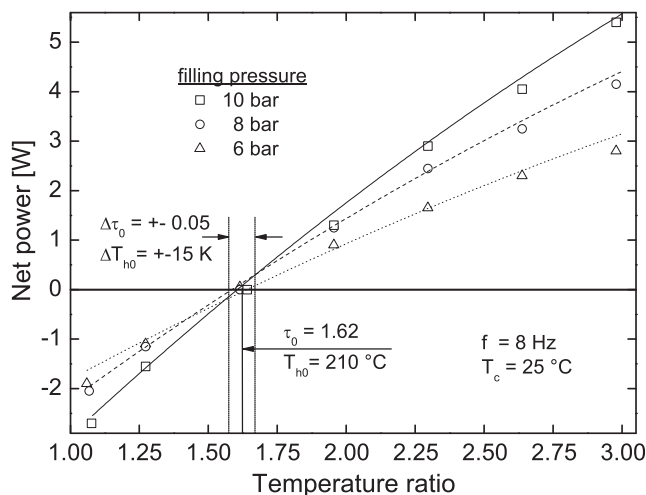


Fig. 17. Fits of the measured net power outputs as functions of the temperature ratio for filling pressures of 6 bar, 8 bar, and 10 bar, an operating frequency of 8 Hz and a cooling temperature of 25 °C.

The gross power P_g of the pulse tube engine is related to the pV -work through $P_g = fW_{pV}$. The net power output of the engine is $P_n = P_g - P_{l,int}$, where the total internal power loss $P_{l,int}$, in general, is a function of at least f, p_f, T , geometry parameters such as r_c, r_d , and V_d as well as working gas properties. The geometry of the engine ($r_c = 0.38, r_d = 1.71$, and $V_d = 18 \text{ cm}^3$) is fixed. Thus, the internal power loss is reduced to being a function of f, p_f, T , and the working gas properties. The main influence of the temperature on the internal power loss is due to the temperature dependency of the working gas properties. Assuming a perfect gas and neglecting the remaining secondary dependency of the internal power loss on the temperature, the internal power loss is only a function of the frequency f and the filling pressure p_f .

The net power can thus be fitted with

$$P_n = f f W_{pV} - P_{l,int}, \quad (67)$$

where f represents a constant fitting parameter.

For a fixed frequency and filling pressure, the factor f and the total internal power loss $P_{l,int}$ can be obtained from the experiment. To do so, the net power output was measured for filling pressures of 6 bar, 8 bar, and 10 bar and frequencies from 4 Hz to 12 Hz. For each couple of (f, p_f) , the engine was motored at heat input temperatures between 40 °C and 600 °C. The minimum pressure is considered to be independent of the temperature distribution. In reality, due to the thermal relaxation, the temperature distribution slightly influences the minimum pressure and the pressure amplitude. The experimentally observed minimum pressures are $p_{min} = 5.15 \text{ bar}$ for a filling pressure of 6 bar, $p_{min} = 7.00 \text{ bar}$ for a filling pressure of 8 bar, and $p_{min} = 8.55 \text{ bar}$ for a filling pressure of 10 bar.

The measured net power P_n is plotted versus the temperature ratio $\tau = T_h/T_c$ and adapted with Eq. (67). The fitting parameters are f and $P_{l,int}$. For $f = 8 \text{ Hz}$, the result is shown in Fig. 17.

Almost independent of the filling pressure, the net power becomes zero for $\tau = 1.62 \pm 0.05$. The respective minimal heat input temperature above which the engine is working is $T_h = 210 \text{ °C} \pm 15 \text{ K}$. The minimal temperature ratio above which the engine provides a gross power output was found with Eq. (61) to be $\tau_0 = (r_d + 1)/r_d$. The distribution ratio of the engine is $r_d = 1.71$ and thus $\tau_0 = 1.59$. This value is within the range of the experimentally observed value and differs from the measured mean value by about 2%. The result is highly interesting, since simplifications such as

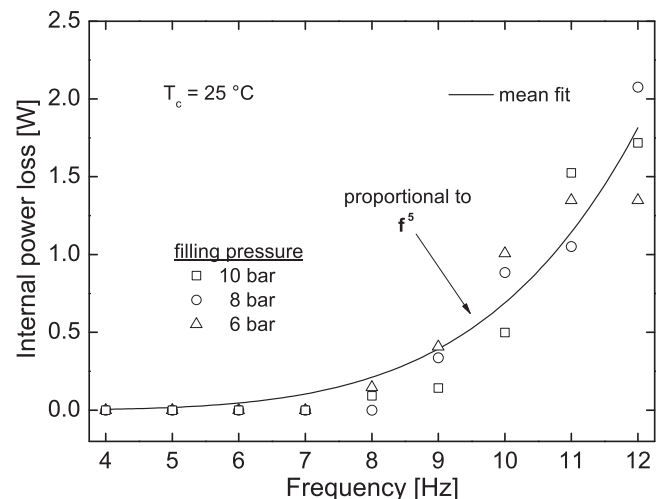


Fig. 18. Internal power losses as functions of the operating frequency for filling pressures of 6 bar, 8 bar, and 10 bar, obtained from the fit of the measured net power in Fig. 17 with Eq. (67). The cooling temperature is 25 °C.

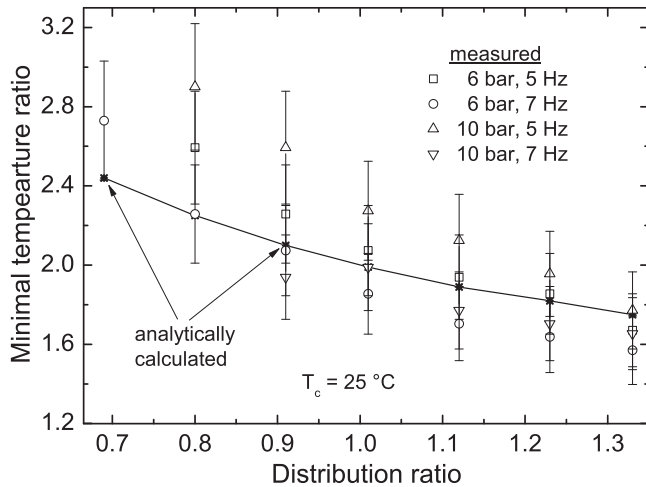


Fig. 19. Comparison of the analytically calculated and the measured minimal temperature ratio above which the engine provides a power output as functions of the distribution ratio. The experiments are done for filling pressures of 6 bar and 10 bar as well as operating frequencies of 5 Hz and 7 Hz. The cooling temperature is 25°C .

isothermal compression and expansion were made for the analytical approach. It seems that the property of the pulse tube engine to have the calculated and measured minimal temperature ratio τ_0 , is a fundamental characteristic of the engine. The analytical model is robust enough to detect this property. The model predicts that the minimal temperature ratio is only a function of the distribution ratio. In the experiment, a weak dependency of τ_0 on the filling pressure was observed.

The fitting factor f is obtained to be $f = 0.159 \pm 0.016$. The internal power loss, also obtained from the fitting procedure, is plotted in Fig. 18. Up to an operating frequency of about 8 Hz, the internal power loss is very low (< 0.5 W). After exceeding this frequency it increases significantly. Fitting the internal power loss with the function $P_{i,int} = c_1 f^l$ delivers an exponent of $l \approx 5$. Mechanical power loss due to deformation of the regenerator mesh is proportional to f and flow friction power loss is proportional to f^3 . Thus, the observed internal power loss does not have its origin in mechanical or flow friction. The thermodynamic situation in the engine seems to change for higher frequencies, leading to the observed strong increase of the internal power loss.

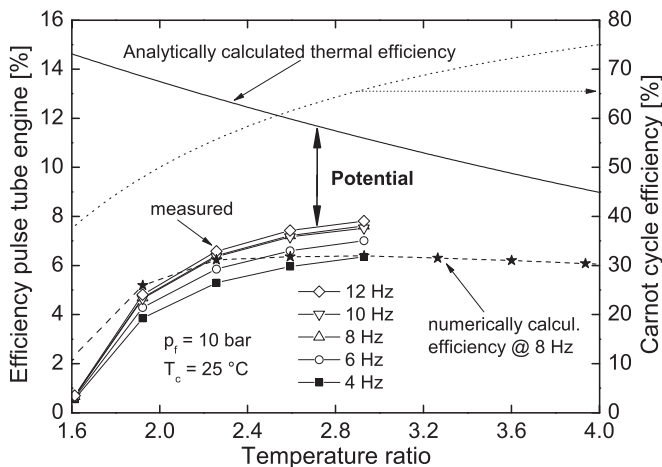


Fig. 20. Comparison of the measured thermal efficiency for operating frequencies between 4 Hz and 12 Hz with the analytically and numerically calculated thermal efficiency as functions of the temperature ratio. The measured efficiencies are available up to a heat input temperature of 600°C . The filling pressure is 10 bar and the cooling temperature is 25°C . The respective Carnot cycle efficiency is also shown.

In order to further investigate the validity of Eq. (61), the size of the regenerator of the working pulse tube engine of [4,24] has been modified. The regenerator is a stack of wire mesh screens having a mesh number of 50, a wire diameter of $200\ \mu\text{m}$, and a screen diameter of 20 mm. The length of the regenerator was varied in steps of 10 mm between 50.5 mm and 110.5 mm resulting in distribution ratios between 0.64 and 1.33 for the 50.5 mm and 110.5 mm regenerator lengths, respectively. For each regenerator size, the minimal temperature ratio above which the engine provides a net power output was measured at filling pressures of 6 bar and 10 bar for operating frequencies of 5 Hz and 7 Hz. According to Fig. 18, the internal power loss at frequencies lower than 8 Hz is negligible, so that $P_g \approx P_n$. For all the distribution ratios investigated, the minimal temperature ratio above which the engine provides a gross power output was calculated with Eq. (61). The results are shown in Fig. 19. The relative error of the measured cooling temperature is estimated to be 1%. Due to the difficulty in the adjustment of the correct heater temperature, its relative error is about 10%. Thus, the error of the measured minimal temperature ratio is about 11%. The error bars are also shown in Fig. 19.

For all filling pressures and frequencies, the analytically calculated minimal temperature ratios are within the range of uncertainty of the measured values. The calculated and measured trends also match. Eq. (61) does not predict a dependency of τ_0 on the filling pressure and the operating frequency, which is not in accordance with the observations in the experiments. As shown in Fig. 19, for a fixed frequency, a higher filling pressure leads to a higher minimal temperature ratio, and for a fixed filling pressure, a higher frequency leads to a lower minimal temperature ratio. For the working engine, the decrease of τ_0 with an increase in the operating frequency is a result of an increase in the phase delay between pressure swing and heat flux in the pulse tube (thermal lag) at higher frequencies. A higher filling pressure on the other hand increases the heat flux and thus decreases the thermal lag. This leads to an increase in τ_0 at higher filling pressures.

In Fig. 20, the measured thermal efficiency of the pulse tube engine is compared with the analytically calculated thermal efficiency according to Eq. (63) in combination with Eqs. (39) and (46) and with the respective Carnot cycle efficiency. The analytically calculated thermal efficiency is an upper limit of the efficiency of the pulse tube engine. It is highest for $\tau = \tau_0$ and decreases with an increase of τ . The measured thermal efficiency of the working pulse tube engine does not show this behavior. It is zero for $\tau = \tau_0$ and increases with an increase of τ . Since the gross efficiency cannot exceed the analytically calculated (ideal) thermal efficiency, it has to decrease above a certain temperature ratio τ_{opt} , which was too high for an experimental investigation in this work. To show the existence of τ_{opt} , the numerical simulation model of the pulse tube engine presented in Ref. [24] was used. The pulse tube engine was simulated for heat input temperatures up to 1000°C ($\tau = 4.27$) and the thermal efficiency is calculated numerically. The result is also shown in Fig. 20. The numerically calculated thermal efficiency reaches its peak value at about $\tau_{opt} \approx 3$. For higher temperature ratios, the thermal efficiency decreases slowly. Further numerical calculations for higher temperature ratios show that the numerically calculated thermal efficiency never reaches the value of the analytically calculated thermal efficiency. For a heat input temperature of 1500°C ($\tau = 5.95$), the numerically calculated thermal efficiency is 5.5%, which is lower than the analytically calculated thermal efficiency of 5.8%. At $\tau = \tau_0$, the numerically calculated thermal efficiency is 2%. This indicates the existence of a finite efficiency at $\tau = \tau_0$ as predicted by the analytical model, but the power output of the working engine for $\tau \rightarrow \tau_0$ is too small to allow an experimental determination of the respective efficiency.

For the working engine, the thermal efficiency increases with higher operating frequencies. The reason is the increase in the phase delay between pressure swing and heat flux in the pulse tube (thermal lag). For higher frequencies, the time at which heat is added to the working gas and removed from the working gas in the pulse tube gets closer to TDC and BDC, respectively. The amount of heat transferred due to thermal relaxation in the pulse tube increases and the cycle gets closer to the theoretical cycle.

Compared to the Carnot cycle efficiency, the efficiency of the pulse tube engine is small. The highest theoretical value of the pulse tube engine's efficiency is for $\tau = \tau_0$ with a value of 14.7% only 40% of the respective Carnot cycle efficiency. Thus, the irreversible nature of the pulse tube engine is clearly evident. Regarding the highest measured efficiency of the working pulse tube engine, its irreversible operation becomes even more obvious. The experimentally achieved efficiency of about 8% at $\tau = 2.9$ amounts to only 12% of the respective Carnot cycle efficiency.

6. Conclusions

The present work represents an analytical approach on the basic characteristics of the pulse tube engine. The main results are summarized as follows:

1. The pV -work of the pulse tube engine can be estimated with Eq. (39).
2. According to Figs. 6 and 9, entropy is always produced if heat is converted into pV -work and vice versa. Thus, the irreversible nature of the pulse tube engine ($\tau > \tau_0$) and the pulse tube heat pump ($\tau < \tau_0$) is proven theoretically.
3. The ideal pulse tube engine starts to operate at a temperature ratio of $\tau \geq \tau_0 > 1$, where according to Eq. (61), $\tau_0 = (r_d + 1)/r_d$ is determined by the distribution ratio. The pV -work output is zero for $\tau = \tau_0$ and increases monotonously for $\tau > \tau_0$, whereas the thermal efficiency has its maximum at $\tau = \tau_0$ and decreases monotonously for $\tau > \tau_0$. This is a result of the strong increase of the entropy production and only a moderately increasing work output. Thus, the pulse tube engine is preferably suited for low temperature ratios τ , which are bound to the condition $\tau > \tau_0$.
4. The theoretically found minimum temperature ratio τ_0 above which the pulse tube engine is working is confirmed experimentally. The measured value amounts to $\tau_0 = 1.62$ ($T_h = 210^\circ\text{C}$), with an absolute error of $\Delta\tau = 0.05$ ($\Delta T_h = 15\text{ K}$). The value calculated with Eq. (61) amounts to $\tau_0 = 1.59$ ($T_h = 210^\circ\text{C}$) and is thus within the error range of the measured value. Since Eq. (61) is obtained from an idealized analysis of the pulse tube engine and the measurement delivers the same result, the minimal temperature ratio τ_0 seems to be a basic property of the pulse tube engine and the developed analytical model is robust enough to determine this basic engine property. Moreover, for different values of r_d , the analytically calculated trend of τ_0 is confirmed experimentally. In contrast to the predictions of Eq. (61), the experimental investigations disclose that τ_0 increases for higher filling pressures and decreases for higher operating frequencies.
5. For the ideal engine, the heat added to the working gas in the cylinder is converted into pV -work. The entire heat added to the working gas in the hot heat exchanger and the pulse tube is rejected in the cold heat exchanger. This effect enables a pulse tube engine with a non-isothermal cylinder to provide a cooling power in the cylinder. The pulse tube engine can thus act as a thermal driven refrigerator, absorbing heat in the cylinder. Even if the thermal energy applied in the hot heat exchanger and the pulse tube is not converted into pV -work, it is essential for the engine to operate.

6. Since heat can be added to the working gas in the pulse tube at temperatures between T_c and T_h , the pulse tube engine is able to use heat sources at different temperatures or absorb heat from a fluid flowing along the pulse tube from its hot to its cold side. It is theoretically possible to design a pulse tube engine, which cools down a fluid, beginning at the hot heat exchanger at the temperature T_h , along the pulse tube to the cold heat exchanger at the temperature T_c . This allows for the removal of a larger amount of heat from a thermal energy carrying fluid compared to e.g. the Stirling engine, which due to its regenerative cycle is only able to use heat at the temperature of the hot heat exchanger T_h .
7. The pV -work and the efficiency of the pulse tube engine are strongly dependent on the distribution ratio. The pV -work output is highest for a certain value of r_d , which can be estimated with Eq. (66) to $r_{d,\text{opt}} \approx (\tau + 1)/(\tau - 1)$. The thermal efficiency decreases monotonously with increasing distribution ratios.
8. An upper limit for the efficiency of the pulse tube engine is found in Eq. (63) and Eq. (64). The measured and numerically calculated gross efficiency never exceeds this limit, which is highest for $\tau = \tau_0$, and decreases monotonously for $\tau > \tau_0$. The gross efficiency of the working engine is zero for $\tau = \tau_0$. It increases up to a certain temperature ratio τ_{opt} , which is found to be about $\tau_{\text{opt}} \approx 3$, and decreases for higher temperature ratios. Due to the irreversible nature of the pulse tube engine, the efficiency of the investigated ideal engine never exceeds 40% of the respective Carnot cycle efficiency. The experimentally achieved highest efficiency [24] is only about 12% of the respective Carnot cycle efficiency.

The listed properties of the pulse tube engine give a pathway for its potential applications and engineering challenges.

Acknowledgments

The author acknowledges the German Federal Environmental Foundation for the financial support of this work within the framework of the PhD Scholarship Program. The author would like to express his gratitude to Tilman Stark and André Rödiger for performing the experiments used in subsection 5.3 as well as to André Thess for stimulating discussions.

References

- [1] Gifford WE, Longworth RC. Pulse tube refrigeration process. In: *Advances in Cryogenic Engineering*, vol. 10. New York: Plenum Press; 1965. pp. 69ff.
- [2] David M, Maréchal JC, Simon Y, Guilpin C. Theory of ideal orifice pulse tube refrigerator. *Cryogenics* 1993;33:154–61.
- [3] de Boehr PCT. Thermodynamic analysis of the basic pulse tube refrigerator. *Cryogenics* 1994;9:699–710.
- [4] Moldenhauer S, Holtmann C, Stark T, Thess A. Theoretical and experimental investigations of the pulse tube engine. *J Thermophys Heat Transf* 2013;27:534–41.
- [5] Chen NCJ, West CD. A single-cylinder valveless heat engine. In: *Proceedings of the 22nd Intersociety Energy Conversion Engineering Conference*, Philadelphia 1987. p. 1386–9.
- [6] Tailor PL. Thermal lag machine. Patent US 5414997; 1995.
- [7] West CD. Some single-piston closed-cycle machines and Peter Tailor's thermal lag engine. In: *Proceedings of the 28th Energy Conversion Engineering Conference*, Atlanta 1993. p. 2673–9.
- [8] Tailor PL. Thermal lag test engines evaluated and compared to equivalent Stirling engines. In: *Proceedings of the 30th Energy Conversion Engineering Conference*, Orlando 1995.
- [9] Hofmann A. Pulsrohr-Leistungsverstärker. Patent DE 10001460A1; 2000.
- [10] Hamaguchi K, Ushijima Y, Hiratsuka Y. Basic characteristics of pulse tube engine. In: *Proceedings of the 12th International Stirling Engine Conference* 2005. p. 275–84.
- [11] Organ AJ. The air engine: Stirling cycle power for a sustainable future. Cambridge: Woodhead Publishing; 2007. p. 107–31.

- [12] Hamaguchi K, Futagi H, Yazaki T, Hiratsuka Y. Measurement of work generation and improvement in performance of a pulse tube engine. *J Power Energy Syst* 2008;2(5):1267–75.
- [13] Yoshida T, Yazaki T, Futaki H, Hamaguchi K, Biwa T. Work flux density measurements in a pulse tube engine. *Appl Phys Lett* 2009;95:044101/1–044101/3.
- [14] Rott N. Thermoacoustics. *Adv Appl Mech* 1980;20.
- [15] Swift GW. Thermoacoustics: A unifying perspective for some engines and refrigerators. New York: Acoustical Society of America, American Institute of Physics Press; 2002.
- [16] Biwa T. Work flow measurements in a thermoacoustic engine. *Cryogenics* 2001;41:305–10.
- [17] Biwa T, Tashiro Y, Mizutani U. Experimental demonstration of thermoacoustic energy conversion in a resonator. *Phys Rev E* 2004;69.
- [18] Backhaus S, Swift GW. New varieties of thermoacoustic engines. In: 9th International Congress on Sound and Vibration 2002.
- [19] Wheatley J, Hofer T, Swift GW, Migliori A. Experiments with an intrinsically irreversible acoustic heat engine. *Phys Rev Lett* 1983;50(7):499–502.
- [20] Ceperly PH. A pistonless Stirling engine – The traveling wave heat engine. *J Acoust Soc Am* 1979;66:1508–13.
- [21] Backhaus S, Tward E, Petach M. Traveling-wave thermoacoustic electric generator. *Appl Phys Lett* 2004;85(6).
- [22] Backhaus S, Swift GW. A thermoacoustic-Stirling heat engine: Detailed study. *J Acoust Soc Am* 2000;107(6):3148–66.
- [23] Moldenhauer S, Thess A, Holtmann C, Fernández-Aballí C. Thermodynamic analysis of a pulse tube engine. *J Energy Convers Manag* 2013;65:810–8.
- [24] Moldenhauer S, Stark T, Holtmann C, Thess A. The pulse tube engine: A numerical and experimental approach on its design, performance, and operating conditions. *Energy* 2013;55:703–15.
- [25] Neveu P, Babo C. A simplified model for pulse tube refrigeration. *Cryogenics* 2000;40:191–201.
- [26] Andersen SK, Carlsen H, Thomsen PG. Control volume based modelling in one space dimension of oscillating, compressible flow in reciprocating machines. *Simul Model Pract Theory* 2006;14:1073–86.
- [27] Bauwens L. Near-isothermal regenerator: Complete thermal characterization. *J Thermophys Heat Transf* 1998;12(3):414–22.
- [28] Gyftopoulos EP, Beretta GP. Thermodynamics. New York: Dover Publications; 2005.
- [29] Thess A. The entropy principle. Berlin: Springer – Verlag; 2011.
- [30] Bronstein IN, Semendjajew KA, Musiol G, Mühlig H. Taschenbuch der Mathematik. Frankfurt am Main: Verlag Harri Deutsch; 2001.